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Vibration of hot-wire anemometer filaments

By A. E. PERRY AND G. L. MORRISON
Department of Mechanical Engineering,
University of Melbourne, Australia

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Hot-wire filament vibration generated by fluctuating aerodynamic loads is found to occur in turbulent flow fields. The motion of the filament relative to the stream can cause errors in the indicated velocity fluctuations. Analysis and experiment show that these errors are probably serious only when the velocity fluctuations are confined within a narrow bandwidth spectrum such as in a Kármán vortex street. Under such conditions the filament goes into a skipping or 'whirling' mode of motion and the errors in the velocity perturbation reading are substantial. Errors in normally encountered boundary-layer turbulence are suspected not to be serious but this is not yet certain. A convenient experimental method for detecting these vibrations is outlined.

A study is made of the effect of filament vibration on the direct frequency-response test for hot-wire systems. These tests include shaking the probe at high frequencies in a steady stream and the Kármán vortex street method.

1. Introduction

The phenomenon of hot-wire vibration was first observed during experiments aimed at determining the frequency response of constant-temperature hot-wire anemometers. Two response tests were used. The first involved the measurement of velocity perturbation profiles across a number of Kármán vortex streets shed from circular cylinders. The Reynolds number of all the cylinders was held constant and a series of different cylinder diameters were used to obtain velocity perturbations of constant amplitude at different frequencies.

The second method involved mounting the hot-wire probe on a vibrating table and oscillating the wire sinusoidally in a steady stream. The hot-wire output was compared with the integrated output from an accelerometer which was mounted with the hot-wire probe.

The first method indicated a flat frequency response ($\pm 1\,\%$) to the highest attainable frequency (10 kHz) provided that the Reynolds number of the cylinders was low (< 150) and that the hot wire was calibrated dynamically using the low-frequency (< 10 Hz) shaking method reported by Perry & Morrison (1971*b*). When the same wire and anemometer system was tested using a high cylinder Reynolds number (> 1000) for the same vortex shedding frequencies and same non-dimensional turbulence levels as for the low Reynolds number case, a gradual fall in the system response with frequency was observed. This amounted to a 25 % decrement at 10 kHz.

The second method revealed low-frequency resonances near 500 Hz. These resonances were due to motion of the hot wire relative to the vibrating table, hot-wire prongs and accelerometer. These unexpected low-frequency resonances corresponded with the beginning of the fall in the system response observed during the high Reynolds number vortex-shedding tests. This coincidence indicated that wire vibration could be occurring during the vortex wake measurements. If wire motion were generated by the vortex perturbations the wire would move with the perturbing force thus causing a decrease in the fluid velocity relative to the hot wire, as was observed.

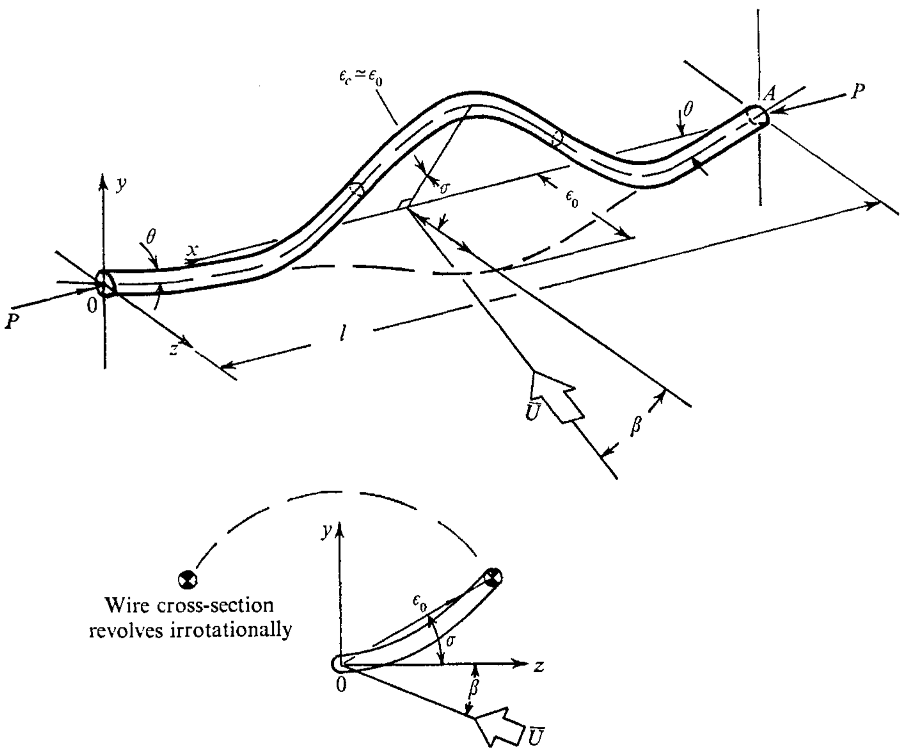

FIGURE 1. Shape of bowed wire when deflected by an air stream.

Although wire vibration was observed experimentally, the positions of the low-frequency resonances were more than an order of magnitude less than the natural frequencies calculated from the assumptions that the wire was an elastic beam built-in at both ends. Even if the wire is assumed to be bent the natural frequency for torsional vibration is one to two orders of magnitude higher than the observed resonances. This paradox was resolved by experimenting with large-scale models of a hot wire. When a real hot wire which is initially straight is heated, it expands and produces an elastic bow and becomes a buckled column with built-in ends. This situation can be modelled by clamping a circular rod at each end while it is elastically bowed by an axial thrust. Such a model is found to have extremely low resistance to lateral loads. If the rod is rotated about axis OA in figure 1 each cross-section revolves irrotationally about the axis, thus no

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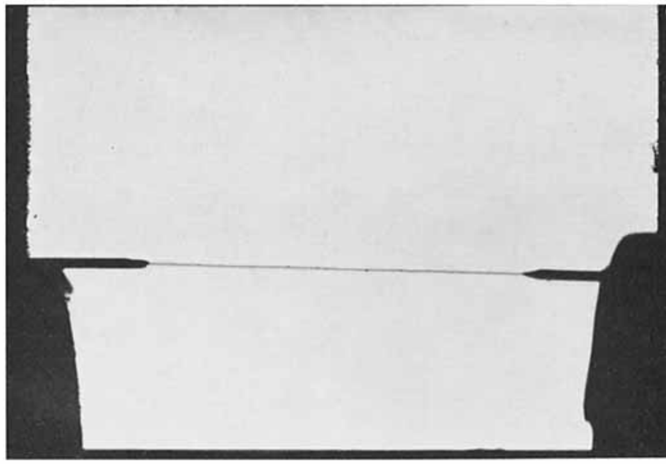

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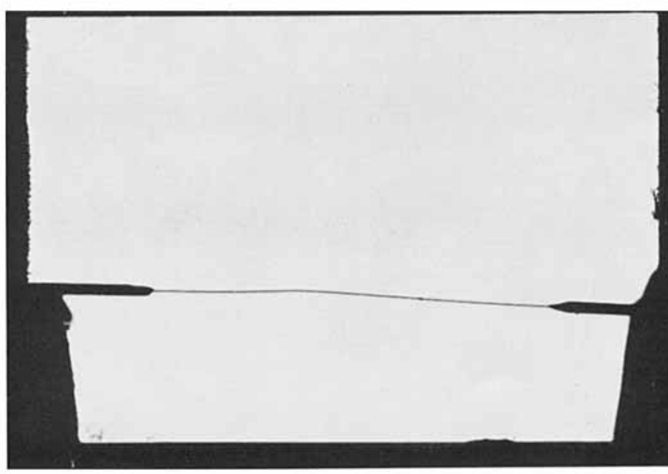

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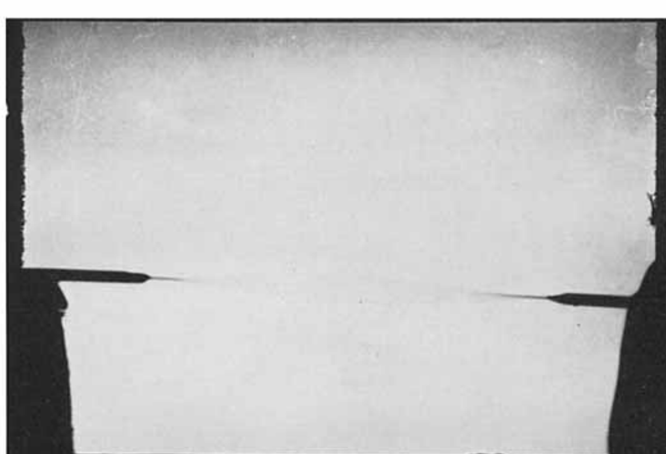

FIGURE 2. (*a*) Wire filament at ambient temperature. (*b*) Wire filament at operating temperature. $R_w = 2{\cdot}0\ R_g$, where R_w is the heated wire resistance and R_g the wire resistance at ambient temperature. (*c*) Wire filament in Kármán vortex wake. $u'/U_\infty = 12\,\%$ r.m.s., $R_w = 2{\cdot}0\ R_g$.

torsion is developed during the motion. If the ends of the rod are perfectly aligned (figure 1, $\theta = 0$) the elastic resistance to rotation is theoretically zero.

Observations of real hot wires shows that the ends are generally misaligned thus providing some resistance to lateral motion. This behaviour would not apply if the rod or hot wire were plastically bent (i.e. permanently bent without end thrust).

An analysis based on the above model of a hot wire was carried out by Perry (1971). For some cases the problem is non-linear and an analog computer solution revealed that in the high Reynolds number Kármán vortex streets it is possible for the hot wire to 'whirl' or 'skip' in a nearly circular orbit about the axis OA with angular frequencies corresponding to the vortex street frequencies.† The errors in the indicated velocity perturbations so computed were found to be of the same order as the observed errors and extended over a wide frequency range. The computed vibration amplitudes were large and indicated that they might be visually observable for a real hot wire. This was confirmed and a photograph of a whirling hot wire (4 μm diameter, 1·6 mm long, Wollaston type platinum wire) is shown in figure 2(*c*) (plate 1). Figures 2(*a*) and 2(*b*) (plate 1) show the unheated and heated wire in ambient air. The elastic bowing that occurs when the wire is heated is evident. It was found that errors in the velocity perturbation readings occurred only when the wire was observed to vibrate, thus discounting other possible reasons for the errors.

These results lead to questions of more general interest in the field of hot-wire measurement. Since hot wires do vibrate in frequency ranges within the energy containing band of turbulence spectra substantial errors could be involved.

2. Results of analysis

The analysis developed so far must be regarded only as a first attempt since it is limited to wires which, when unloaded laterally, are bowed symmetrically with the bow contained in a plane. Large-scale models of wires bowed asymmetrically and those bowed in three dimensions were cursorally studied and found to exhibit the same important features as the simple model outlined here.

Most hot wires are elastically bowed because of thermal expansion. The model adopted here is a circular rod built-in at both ends and subjected to an axial thrust P which is generated by the thermal expansion of the wire (figure 1). The rod has end angles θ at the supports and is loaded by an airstream of mean velocity $\overline{U}$ which is inclined at an angle β to the xz plane. The loading causes the wire to deflect in a circular arc through an angle σ. If $2\epsilon_0/l\theta > 2{\cdot}5$ the wire rotates approximately as a plane figure without change of shape about the axis OA. If the flow has turbulent velocity components u' and v' parallel and transverse to the mean flow, analysis of the above model shows that the angular position of the wire is given by the following relation for $u'/\overline{U}$ and $v'/\overline{U} < 0{\cdot}15$.

$$\sigma'' + \chi[1+(1+Q)\sin^2(\beta+\sigma)]\sigma' + \sin\sigma - \xi\sin(\beta+\sigma)$$
$$= \xi(2+Q)\sin(\beta+\sigma)u'/\overline{U} + \xi\cos(\beta+\sigma)v'/\overline{U}, \quad (1)$$

† A full account of the analysis, linearized solutions and the non-linear analog computer solutions is given by Perry (1971). The analysis is valid only for constant-temperature hot wires.

where $$\xi = \frac{\rho a \overline{U}^2 C_D l^3}{16 E I \theta \pi^2}, \quad \chi = \frac{6^{\frac{1}{2}}}{16}\rho a \overline{U} C_D \left[\frac{\epsilon_0 l^3}{\gamma E I \theta \pi^2}\right]^{\frac{1}{2}}$$

and $$Q = d(\log C_D)/d(\log Re).$$

The primes on σ denote differentiation with respect to a non-dimensional time τ given by
$$\tau = t\left[\frac{32\theta\pi^2 EI}{3\gamma\epsilon_0 l^3}\right]^{\frac{1}{2}}$$

and t is real time. Other variables are as follows: γ is the mass per unit length of wire; ρ the fluid density, l the distance between the ends of the wire, E Young's modulus of elasticity of the wire material, I the second moment of area of the wire cross-section taken about a diameter; a the wire diameter and C_D and Re are the wire drag coefficient and Reynolds number respectively.

Equation (1) is valid for arbitrarily large σ. If $u'/\overline{U}$ and $v'/\overline{U}$ are less than 0·15 the hot wire will sense the components of relative velocity parallel to the $\overline{U}$ direction only. If u'_e is the velocity fluctuation registered by the hot wire and u' is the actual velocity fluctuation, then it can be shown that

$$u'_e/\overline{U} = u'/\overline{U} - \tfrac{2}{3}(\chi/\xi)\sigma' \sin(\beta+\sigma). \tag{2}$$

Equations (1) and (2) are completely characterized by the parameters χ, ξ and β. In principle they can be solved, given the forcing terms $u'/\overline{U}$ and $v'/\overline{U}$ as functions of time. The mean square ratio $\overline{u_e'^2}/\overline{u'^2}$ can then be determined. Ideally this quantity should be equal to unity but will depart from this if wire vibration is serious. Analytical solutions to the linearized approximations of (1) and (2) can be obtained and one is shown in figure 3 (a). Measured spectra of boundary-layer turbulence will be in error by the amount shown provided $u'/\overline{U}$ is sufficiently small for the linearity assumption to be valid. The resonance frequency f_R in Hz is given by
$$f_R = \frac{1}{2\pi}[\xi^2 - 2\xi\cos\beta + 1]^{\frac{1}{4}} \bigg/ \left(\frac{3\gamma\epsilon_0 l^3}{32 E I \theta \pi^2}\right)^{\frac{1}{2}}. \tag{3}$$

Calculated values of f_R were of the order of 1 kHz and this agreed with the high-frequency shaking tests described in §4.3. The quantity M shown in figure 3 can depart appreciably from unity and theoretically can have a value as low as $\frac{1}{9}$. However, this need not present a problem if the bandwidth of the error is sufficiently small. By way of example, the authors took a 'standard wire' as being made from platinum 1·6 mm long, 4 μm in diameter and with end angles of $\theta = 0{\cdot}02$ rad. The specific gravity of platinum is 22 and $E = 1{\cdot}5 \times 10^{11}\,\mathrm{N/m^2}$. ϵ_0/l was chosen to be 0·1 since this appeared to be typical from examinations of the authors' heated wires. For air at ambient conditions or even at film temperature conditions, with the velocity $\overline{U}$ in the range 3 to 30 m/sec, the bandwidth was negligible for all β except for a very narrow velocity range. In practice θ is never known precisely. By varying θ no case could be found which would give the dangerous situation of both small M and an appreciable bandwidth except at very low $\overline{U}$ ($<$ 1 m/sec). The component of fluid velocity v' also causes wire oscillation. However, according to the linear theory, its effect on $\overline{u_e'^2}/\overline{u'^2}$ is an order of magnitude smaller than the influence of u' except under conditions explained in §4.3.

From this discussion it seems more likely that serious errors occur because of the non-linearities in (1) and (2).

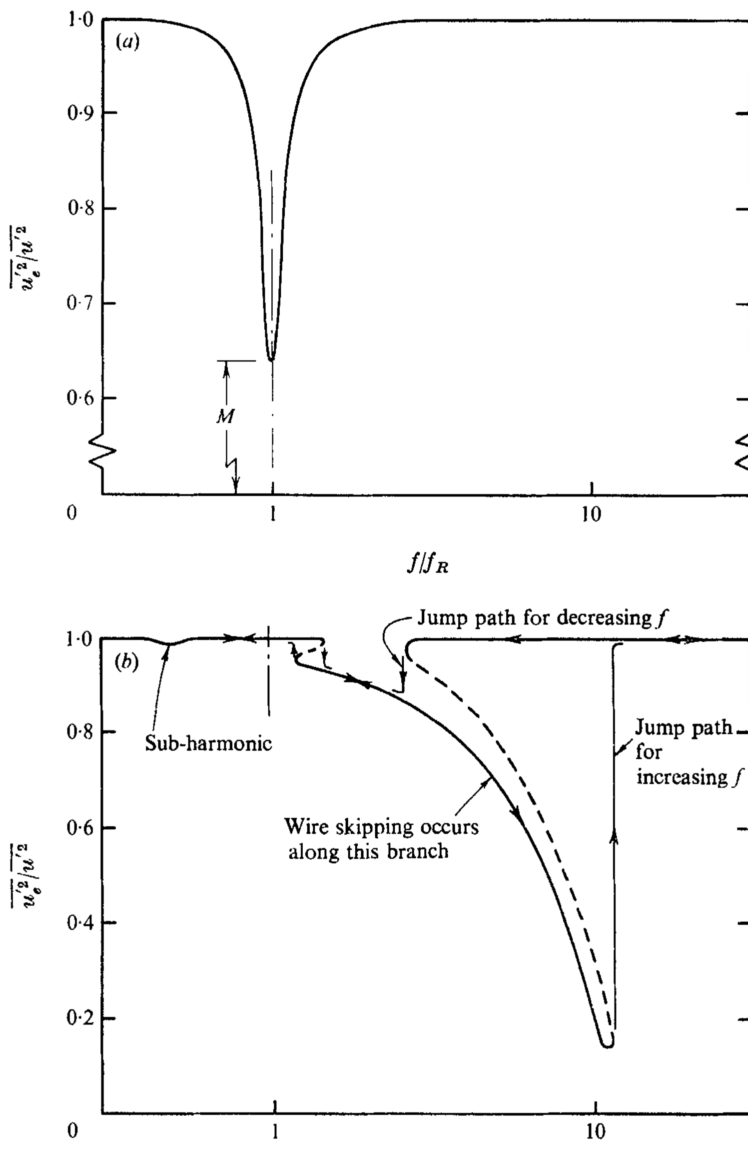

FIGURE 3. Solutions to equations (1) and (2) with u' fluctuations alone. (*a*) Linearized solutions. Bandwidth of error shown is the maximum encountered in test calculations. (*b*) Non-linear solution obtained on analog computer. $u'/U' = 10\,\%$ r.m.s.; – – –, conjectural unstable solutions. Note the multivalued solutions and the 'hysteresis effect' of increasing and decreasing the frequency.

3. Non-linear behaviour

Equations (1) and (2) were studied on an analog computer using a C_D versus R_e function obtained from Tritton (1959). Sinusoidal signals were used for u'. For the standard wire the linear solution was applicable for low mean velocities (< 10 m/sec) but for higher velocities the linearity assumption broke down for velocity perturbation levels as low as 1 % r.m.s., because the amplitude of oscillation of the wire became large. For certain velocity ranges the wire tended to spin about its axis and the angle σ thus became unbounded.

Figure 3 (*b*) shows a typical $\overline{u_e'^2}/\overline{u'^2}$ versus frequency plot for the standard wire at 18 m/sec and $\beta = 0{\cdot}1$ rad when subjected to u' perturbations alone. Note the non-linear jump phenomenon.

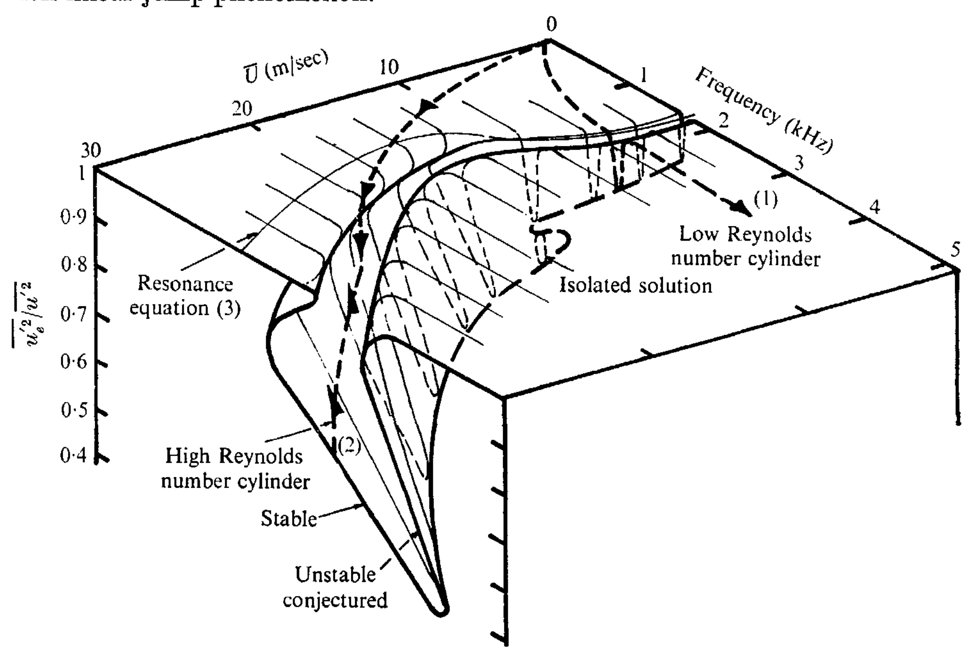

FIGURE 4. Non-linear wire-vibration Bode diagram for effective velocity ($u'/\bar{U} = 10$ % r.m.s., $\beta = 0{\cdot}1$ rad). With v' perturbation present results were much the same.

The phenomenon of wire skipping explains why a fall off in $(\overline{u_e'^2})^{\frac{1}{2}}/\bar{U}$ was observed in the high Reynolds number vortex-shedding measurements and why no fall off was observed during the low Reynolds number tests. By repeating the solution for different mean velocities a three-dimensional surface for $\overline{u_e'^2}/\overline{u'^2}$ was mapped out and is shown in figure 4 for $u'/\bar{U} = 10$ % r.m.s. The intricate jumps and convolutions are not shown but the more important large variations which produce substantial errors are shown. The trajectories (1) and (2) in figure 4 are parabolas in the $f\bar{U}$ plane and represent the end states of the vortex shedding experiments since for vortex shedding from circular cylinders $f \propto \bar{U}^2$ if the cylinder Reynolds number is kept fixed. These experiments are described in §4. Curve (1) is the low Reynolds number case which traverses the sharp 'linear groove' only. The effect of this was unnoticed during the experiments because of the coarseness of the frequency steps used. Curve (2) corresponds to the high Reynolds number case and at high frequencies or high mean velocities the solution trajectory 'slips' into the large sloping groove and follows it. Under these conditions the wire is skipping. The same general trends were noted for $\beta = \frac{1}{2}\pi$ and $0{\cdot}9\pi$.

The effect of the bandwidth of a random u' signal was investigated in the frequency range where wire skipping was observed. From cursory tests on the analog computer it was found that, although the wire skipped erratically in one direction and then the other for a bandwidth of about 2 octaves, errors in u_e' were small until the bandwidth was reduced to one octave. The $u'/\overline{U}$ signal was 10 % r.m.s. and was obtained by passing white noise through a variable notch filter. All vortex shedding experiments had bandwidths much less than an octave. Normally encountered turbulence has, of course, a much broader bandwidth. Whether the favourable result with turbulence is universally valid will not be known until more cases and more complicated wire geometries are studied. However the authors suspect that the linear theory is valid for broad band turbulence in most cases.

4. Experimental observations

4.1. *Kármán vortex street tests*

The velocity perturbation profiles across the wake of a circular cylinder were measured. These perturbations are given by

$$(\overline{u'^2})^{\frac{1}{2}}/U_\infty = \phi_1[Re, x/d, y/d], \tag{4}$$

where U_∞ is the free-stream velocity, Re the cylinder Reynolds number, x/d the number of cylinder diameters downstream from the cylinder and y/d the number of cylinder diameters off the wake centre-line. The cylinder Strouhal number is

$$St = fd/U_\infty = \phi_2[Re], \tag{5}$$

where f is the frequency of vortex shedding from one side of the cylinder.

If a range of cylinder diameters is used and Re is kept constant, the $(\overline{u'^2})^{\frac{1}{2}}/U_\infty$ versus y/d profiles should be identical at a fixed x/d. This should be true even though (4) and (5) show that the frequency f is proportional to U_∞^2 for constant Re.

The most convenient Re range is 50 to 150 because then the wake is laminar and a nearly pure sinusoidal velocity perturbation is generated. The results of such a test at $Re = 140$ are shown in figure 5 and demonstrate that the frequency response was flat to beyond 10 kHz. Higher frequencies could not be obtained with the available equipment. For $Re > 350$ the vortices are turbulent thus the velocity perturbations cover a narrow band of frequencies centred at the vortex shedding frequency. The profile measurements were repeated for $Re = 2000$ to investigate the response of the system at high bias velocities. The free-stream speed in the $Re = 140$ tests was $1{\cdot}5 < U_\infty < 10$ m/sec, while for the $Re = 2000$ tests the speeds were in the range $10 < U_\infty < 40$ m/sec. The results for $Re = 2000$ using the same hot wire and feedback system as in the $Re = 140$ tests are shown in

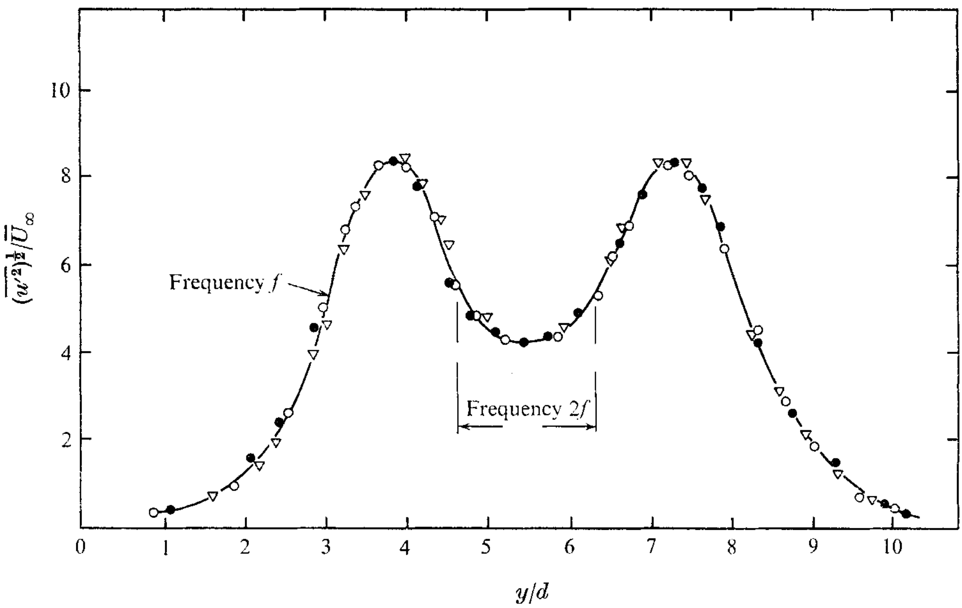

FIGURE 5. Kármán vortex-wake velocity-perturbation profiles for $Re = 140$. Origin for y is arbitrary. $x/d = 20$; ∇, frequency $f = 500$ Hz; $\bigcirc$, 1500 Hz; $\bullet$, 4000 Hz.

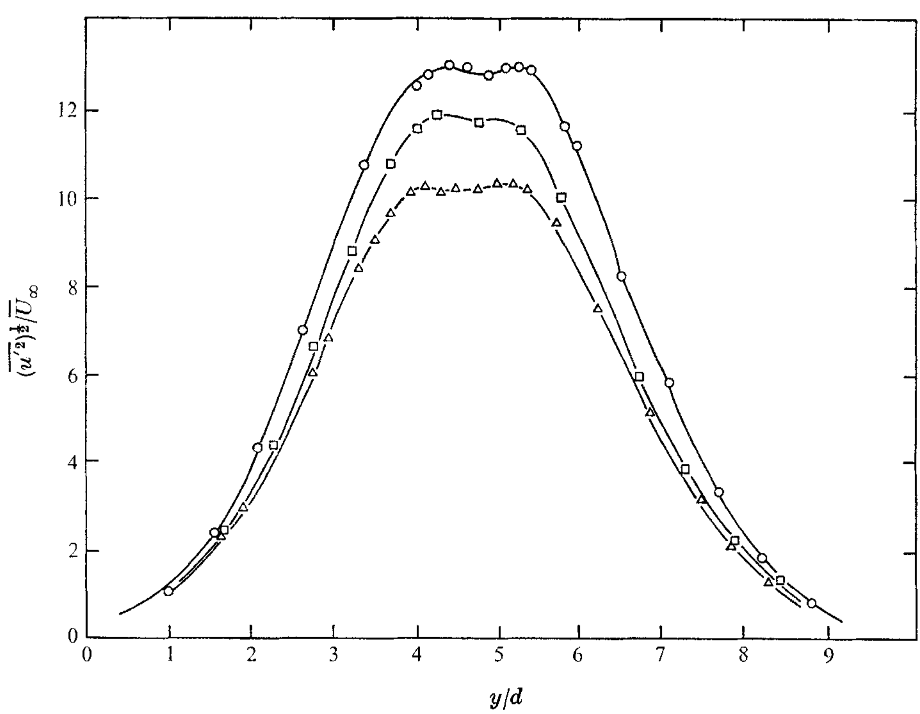

FIGURE 6. Kármán vortex-wake velocity-perturbation profiles for $Re = 2000$. Origin for y is arbitrary. $x/d = 14$; $\bigcirc$, frequency = 540 Hz; $\square$, 3100 Hz; $\triangle$, 6000 Hz.

figure 6. These results indicate a 25 % drop at 10 kHz whereas the $Re = 140$ tests were flat to beyond 10 kHz. By viewing the hot wire through a microscope it was observed that it was skipping in the $Re = 2000$ tests but not in the $Re = 140$ tests. This behaviour is explained in figure 4.

4.2. *Technique for detecting wire vibration*

A possible means of altering the vibration characteristics of a given hot wire, and thus determining whether wire vibration is affecting the measurement, is to change the wire temperature and thus change its bowing ϵ_0.

If a wire is initially straight and unstrained at ambient temperature the bowing at its centre is proportional to $(\Delta T)^{\frac{1}{2}}$, where ΔT is the wire temperature change. It is difficult to predict the effect of a change of bowing from the non-linear equations but this was tested experimentally on a wire which was known to 'skip' during the high Reynolds number tests. The wire was calibrated at a number of wire

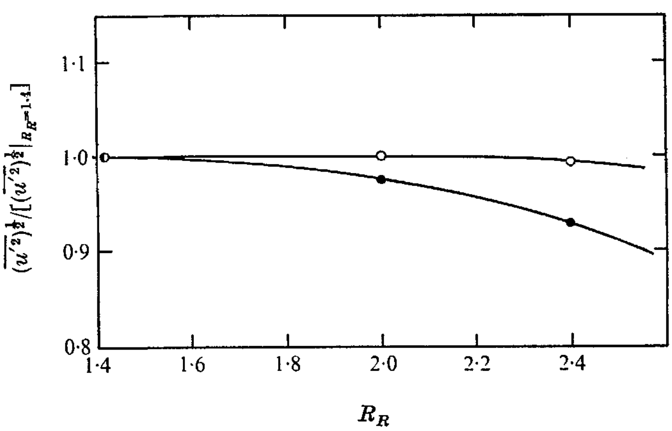

FIGURE 7. Effect of wire temperature on vibration errors (dynamic calibration.) $R_R = R_w/R_g$; ○, $Re = 140$ (no vibration); ●, $Re = 2000$ (vibration present).

temperatures and each calibration was used to evaluate the velocity indicated by the vibrating hot wire for a high free-stream velocity. The variation of the reading versus wire resistance ratio R_R is shown in figure 7. This figure also shows the results of an experiment performed on the same hot wire at a low free-stream velocity (no vibration observed) and at the same vortex-shedding frequency. The reading was the same for all wire temperatures (1 % low at $R_R = 2{\cdot}4$). This is a convenient test for determining whether or not wire vibration is occurring in a given situation. However, one must be careful to make suitable adjustments to the anemometer system to ensure optimum response and stability at the various operating points. (See Perry & Morrison 1971 *a*.) Also it should be stressed that the wire should be dynamically calibrated as described by Perry & Morrison (1971 *b*). The usual static calibration techniques give scattered results.

The above test is suitable for narrow band signals. For broad band signals a suitable test is suggested by equation (3). If the turbulence spectrum is measured then serious vibration would show up as a noticable dip centred at frequency f_R.

By varying the wire resistance ratio, ϵ_0 will alter and the resultant variations in f_R will cause the dip to shift. However this test will be valid only if the linear theory is applicable. Problems with wire vibration might be avoided by using two different hot wires and rejecting the data if the results do not agree. However, from the authors' experience with the vortex-shedding test it was found that fair agreement between two wires quite often occurred, even though both results were known to be in error. The other tests are more convenient and can be made systematic.

4.3. *High-frequency shaking tests*

A word of warning is appropriate regarding the direct testing of frequency response of hot-wire anemometers by shaking the wire probe at high frequencies in a steady air stream. Attempts have been made by comparing the u'_e signal with the integrated output from an accelerometer mounted with the probe on a vibrating table. Analysis for this case, based on a modification of equations (1) and (2) to account for the extra inertia terms, shows that beyond resonance u'_e becomes as sensitive to v' as it is to u'. This has been verified by experiment and the results of such tests become intractable. It is difficult to remove the unwanted v' motions of the vibrating table at high frequencies.

5. Discussion and conclusions

From this investigation one would conclude that hot-wire vibration becomes a problem only if the spectrum of velocity perturbation is narrow. However, it should be emphasized that only a limited range of the parameters ξ, χ and β were studied on the analog computer. A more detailed investigation was not possible with the facilities available, thus there may be cases not investigated here where vibration introduces errors for broad band signals. To fully understand hot-wire vibration phenomena a more detailed study of the equations presented here, or extensions thereof for the full range of parameters χ, ξ and β with broad and narrow band signals, is necessary.

For the case investigated experimentally, vibrations were detected by operating the wire at different temperatures and observing a change of the indicated velocity fluctuations. Owing to the complicated form of the wire's behaviour this test may not always be conclusive. There could be cases where vibration is present but wire temperature changes have negligible effect.

The experiments with real hot wires showed that the resonant frequency is of the order predicted by the model. Because of the wire's vibration, it is difficult to obtain meaningful results for the frequency responses of a hot-wire system by shaking the wire at high frequency. The vortex-shedding experiments verify that hot wires do vibrate and give large errors for narrow band signals. If vortex-wake measurements are to be used for verifying hot-wire system frequency responses, the Reynolds number of the vortex-shedding cylinder, or more specifically, the mean drag force on the wire must be kept low.

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CORRIGENDUM

'Static and dynamic calibration of constant-temperature hot-wire systems',

by A. E. PERRY AND G. L. MORRISON,

J. Fluid. Mech. vol. **47**, 1971, pp. 765–777

The authors should have pointed out that the relatively low frequency response of the Disa 55A01 hot-wire set occurs only at speeds less than 10 m/sec, and that this response is not typical of higher velocity operation. The authors' low-speed results are consistent with an extrapolation of Disa's specifications into the range tested. The Disa 55D01 supersedes the above model and the manufacturers claim good performance at all speeds for this newer system.